Measurement-Induced-Nonlocality via the Unruh effect

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Abstract

Treated beyond the single-mode approximation, Measurement-Induced-Nonlocality (MIN) is investigated for both Dirac and Bosonic fields in non-inertial frames. Two distinctly differences between the Dirac and Bosonic fields are: (i) the MIN for Dirac fields persists for any acceleration, while the quantity for Bosonic fields does decay to zero in the infinite acceleration limit; (ii) the dynamic behaviors of the MIN for Dirac fields is quite different from the Bosonic fields case. Besides, we also study the nonlocality for Dirac fields and find that the MIN is more general than the quantum nonlocality related to violation of Bell's inequalities. Meanwhile some discussions of geometric discord are presented too.

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I. INTRODUCTION

The investigation of relativistic quantum information not only supplies the gap of interdiscipline refer to quantum information and relativity theory, but also has a positive promotion on the development of them. As a result of that, this domain has been paid much attention in the last decade [1–15]. Among them, most papers have focused on quantum resource, e. g., quantum entanglement [1–6, 9, 15] and discord [11, 12], because the quantum resource plays an very important role in the quantum information tasks such as teleportation [16] and computation [17, 18], and studying it in a relativistic setting is very closely related to the implementation of quantum tasks with observers in arbitrary relative motion. In addition, extending this work to the black hole background is very helpful for us to understand the entropy and paradox [19, 20] of the black hole.

Despite much effort has been paid to extend quantum information theory to the relativistic setting, most papers are limited to entanglement and discord, another foundation of quantum mechanics–nonlocality is barely considered. Recently, Nicolai Friis *et al* firstly studied the nonlocality in the noninertial frame, and they pointed out that residual entanglement of accelerated fermions is not nonlocal [21]. Following them, Alexander Smith *et al* studied the tripartite nonlocality in the noninertial frames [22], and DaeKil Park considered tripartite entanglement-dependence of tripartite nonlocality [23]. Generally, most researchers analyzed the quantum nonlocality by means of Bell's inequalities [24] for bipartite system and Svetlichny inequality for tripartite system [25], respectively. Because, these inequalities are satisfied by any local hidden variable theory, but they may be violated by quantum mechanics. However, Shunlong Luo and Shuangshuang Fu have introduced a new way to quantify nonlocality by measurement, which is called the MIN [26], and following their treatise, a number of papers emerged to perfect its definition [27, 28] and discussed its properties [29, 30]. In addition, some authors have analyzed its dynamical behavior and compared it with other quantum correlation measurements such as the geometric discord [31, 32]. However, all of these studies don't involve the effect on the MIN resulting from relativistic effect. In fact, the study that how the Unruh effect [33] affects the MIN can help us implement the quantum task better and more efficient. Inspired by that, in this paper we analyze how the Unruh effect affects the MIN for both the Dirac and Bosonic fields and find some new properties.

Our paper is constructed as follows. In section II we simply introduce the definition of the MIN. In sections III and IV how the Unruh effect affects the MIN for Dirac and Bosonic fields is respectively studied. And we summarize and discuss our conclusions in the last section.

II. DEFINITION OF MEASUREMENT-INDUCED-NONLOCALITY

Recently, Luo *et al* [26] have introduced a way to quantify nonlocality from a geometric perspective in terms of measurements, which is called the MIN. For a bipartite quantum state ρ shared by subsystem A and B with respective system Hilbert space H^A and H^B, we can find the difference between the overall pre- and post-measurement states by performing a local von Neumann measurements on part A. To capture the genuine nonlocal effect of measurements on the state, the key point is that the measurements do not disturb the local state $\rho^A = tr_B\rho$. Based on this idea, the MIN can be defined by

$$N(\rho) = \max_{\Pi^A} \parallel \rho - \Pi^A(\rho) \parallel^2 . \tag{1}$$

For a general 2×2 dimensional system

$$\rho = \frac{1}{2}\frac{\mathbf{1}^A}{\sqrt{2}} \otimes \frac{\mathbf{1}^B}{\sqrt{2}} + \sum_{i=1}^{3} x_i X_i \otimes \frac{\mathbf{1}^B}{\sqrt{2}} + \frac{\mathbf{1}^A}{\sqrt{2}} \otimes \sum_{j=1}^{3} y_j Y_j + \sum_{i=1}^{3}\sum_{j=1}^{3} t_{ij} X_i \otimes Y_j, \tag{2}$$

its MIN is given by [26]

$$N(\rho) = \begin{cases} trTT^t - \frac{1}{\|\mathbf{x}\|^2}\mathbf{x}^t TT^t \mathbf{x} & \text{if } \ \mathbf{x} \neq 0, \\ trTT^t - \lambda_3 & \text{if } \ \mathbf{x} = 0, \end{cases} \tag{3}$$

where $TT^t(T = (t_{ij}))$ is a 3×3 dimensional matrix, λ_3 is its minimum eigenvalue, and $\|\mathbf{x}\|^2 = \sum_i x_i^2$ with $\mathbf{x} = (x_1, x_2, x_3)^t$.

III. MIN FOR DIRAC FIELDS

Two sets of modes named Rindler and Minkowski modes are corresponding to the solutions of Dirac equation in Rindler and Minkowski space-time, respectively. It is found that the combinations of Minkowski modes called Unruh modes can be transformed into monochromatic Rindler modes. Besides, they share the same vacuum state as the standard

monochromatic Minkowski modes. Therefore, the annihilation operators of Unruh modes can also annihilate the Minkowski vacuum, and they are give by

$$C_{\Omega,R/L}=\frac{1}{(e^{\Omega/T}+1)^{1/2}}(e^{\Omega/2T}c_{\Omega,I/II}-d^{\dagger}_{\Omega,II/I}), \tag{4}$$

where $c^{\sigma}_{\Omega,\varsigma}$ and $d^{\sigma}_{\Omega,\varsigma}$ with $\sigma=\{\dagger,-\}$ and $\varsigma=\{I,II\}$ denote that particle and antiparticle operators in region ς, respectively, and T is Unruh temperature.

Going beyond the single mode approximation [8], we consider a general Unruh mode being an arbitrary combination of the independent Unruh modes (4) such as

$$c^{\dagger}_{\Omega,U}=q_L(C^{\dagger}_{\Omega,L}\otimes\mathbf{1}_R)+q_R(\mathbf{1}_L\otimes C^{\dagger}_{\Omega,R}). \tag{5}$$

By using the creation operator to act on the vacuum, the associated single particle state is easily obtained.

It is needed to note that the Unruh modes are spatial delocalized, so can not be completely measured. Additionally near the acceleration horizon they are highly oscillatory from the inertial observer's viewpoint. This implies that they are physically unfeasible states. However, using the Unruh modes, we can get a family of solutions only depending on the parameterized acceleration r (or the Unruh temperature T). The entangled state constructed by them will be the maximally entangled in the inertial limit, and from the perspective of accelerated observers it is single-frequency entangled state. Moreover, as argued in Ref. [34], the use of Unruh modes to extract conclusions regarding fundamental physics can be justified according to some desirable features, such as they are purely positive frequency linear combinations of Minkowski modes, and share the same vacuum with Minkowski monochromatic modes, and so on. These features ensure that the parameterized acceleration dependent states can describe the behavior of maximally entangled states in small acceleration limit, and they also yield the correct behavior for physical states in the large acceleration limit, so a main result (different behaviors between the Bosonic fields and Dirac fields in the infinite accelerated limit) in our paper can also applies to the physical states. Furthermore, such results qualitatively describe the behavior for the relevant states as a function of the acceleration. For these reasons, the Unruh modes are commonly used in literatures.

Usually, the Unruh monochromatic mode $|0_\Omega\rangle_{\mathrm{U}}$, from the non-inertial observers' perspec-

tive, can be expressed as [9, 10]

$$|0_\Omega\rangle_{\mathrm{U}} = \frac{1}{e^{\Omega/T}+1}\left(e^{\Omega/T}|0000\rangle_\Omega - e^{\Omega/2T}|0011\rangle_\Omega + e^{\Omega/2T}|1100\rangle_\Omega - |1111\rangle_\Omega\right), \tag{6}$$

where the notation $|p_\Omega\rangle_I^\dagger |q_\Omega\rangle_{II}^- |m_\Omega\rangle_I^- |q_\Omega\rangle_{II}^\dagger$ is used. Likewise, the particle state of Unruh mode Ω in the Rindler basis is found to be

$$\begin{aligned} |1_\Omega\rangle_{\mathrm{U}} = & \frac{q_R}{(e^{\Omega/T}+1)^{1/2}}\left(e^{\Omega/2T}|1000\rangle_\Omega - |1011\rangle_\Omega\right) \\ & + \frac{q_L}{(e^{\Omega/T}+1)^{1/2}}\left(e^{\Omega/2T}|0001\rangle_\Omega + |1101\rangle_\Omega\right). \end{aligned} \tag{7}$$

Noting that different ordering of operators leads to Fermionic entanglement ambiguity in non-inertial frames because of the anticommutation relation satisfied by Fermionic operators. However, only one of these ordering gives physical results, as done in ref. [10], we adopt the physical ordering in our paper.

A. MIN shared by Alice and Rob

We now assume that Alice and Rob share a X-type initial state

$$\rho_{AB} = \frac{1}{4}\left(I_{AB} + \sum_{i=1}^{3} c_i \sigma_i^{(A)} \otimes \sigma_i^{(B)}\right), \tag{8}$$

where $I_{A(B)}$ is the identity operator in subspace $A(B)$, and $\sigma_i^{(n)}$ is the Pauli operator in direction i acting on the subspace $n = A, B$, $c_i \in \Re$ such that $0 \leq | c_i | \leq 1$ for $i = 1, 2, 3$. Obviously, Eq. (8) represents a class of states including the well-known initial states, such as the Werner initial state ($|c_1| = |c_2| = |c_3| = \alpha$), and Bell basis state ($|c_1| = |c_2| = |c_3| = 1$).

After the coincidence of Alice and Rob, Alice stays stationary while Rob moves with an uniform acceleration a. To describe the states shared by these two relatively accelerated observers in detail, we must use Eqs. (6) and (7) to rewrite Eq. (8) in terms of Minkowski modes for Alice, Rindler modes I for Rob and Rindler modes II for Anti-Rob, which implies that Rob and Anti-Rob are respectively confined in region I and II. The regions I and II are causally disconnected, and the information which is physically accessible to the observers is encoded in the Minkowski modes A and Rindler modes I, but the physically unaccessible information is encoded in the Minkowski modes A and Rindler modes II. So we must trace over the Rindler modes II (modes I) when we only consider the physically accessible (unaccessible) information.

We first consider the MIN between modes A and particle modes in region I. By taking the trace over the states of region II and antiparticle states of region I, we obtain the reduced density matrix of Alice-Rob modes

$$\rho_{A,I}=\frac{1}{4}\begin{pmatrix}\frac{1+q_L^2+c_3q_R^2}{e^{-\Omega/T}+1} & 0 & 0 & \frac{(c_1-c_2)q_R}{(e^{-\Omega/T}+1)^{\frac{1}{2}}}\\ 0 & (1-c_3)q_R^2+\frac{1+c_3q_R^2+q_L^2}{e^{\Omega/T}+1} & \frac{(c_1+c_2)q_R}{(e^{-\Omega/T}+1)^{\frac{1}{2}}} & 0\\ 0 & \frac{(c_1+c_2)q_R}{(e^{-\Omega/T}+1)^{\frac{1}{2}}} & \frac{1-c_3q_R^2+q_L^2}{e^{-\Omega/T}+1} & 0\\ \frac{(c_1-c_2)q_R}{(e^{-\Omega/T}+1)^{\frac{1}{2}}} & 0 & 0 & (1+c_3)q_R^2+\frac{1-c_3q_R^2+q_L^2}{e^{\Omega/T}+1}\end{pmatrix},$$

where $|mn\rangle=|m\rangle_{\rm A}|n_\Omega\rangle_{\rm I}^{\dagger}$. For convenience to calculate the MIN, we rewrite the state $\rho_{A,\rm I}$ in terms of Bloch representation, which is given by

$$\rho_{A,I}=\frac{1}{4}\left(\mathbf{1}_A\otimes\mathbf{1}_I+c_0'\mathbf{1}_A\otimes\sigma_3^{(I)}+\sum_{i=1}^{3}c_i'\sigma_i^{(A)}\otimes\sigma_i^{(I)}\right), \tag{9}$$

where $c_0'=\frac{q_L^2e^{\Omega/T}-1}{(e^{\Omega/T}+1)}$, $c_1'=\frac{c_1q_R}{(e^{-\Omega/T}+1)^{\frac{1}{2}}}$, $c_2'=\frac{c_2q_R}{(e^{-\Omega/T}+1)^{\frac{1}{2}}}$ and $c_3'=\frac{c_3q_R^2}{(e^{-\Omega/T}+1)}$. From Eq. (3), it is easy to get the MIN for the state $\rho_{A,I}$

$$\begin{aligned}N(\rho_{A,I})=\frac{1}{4}\{&\frac{(c_1q_R)^2}{(e^{-\Omega/T}+1)}+\frac{(c_2q_R)^2}{(e^{-\Omega/T}+1)}+\frac{(c_3q_R^2)^2}{(e^{-\Omega/T}+1)^2}\\&-\min[\frac{(c_1q_R)^2}{(e^{-\Omega/T}+1)},\frac{(c_2q_R)^2}{(e^{-\Omega/T}+1)},\frac{(c_3q_R^2)^2}{(e^{-\Omega/T}+1)^2}]\}.\end{aligned} \tag{10}$$

Obviously, $\min[\frac{(c_1q_R)^2}{(e^{-\Omega/T}+1)},\frac{(c_2q_R)^2}{(e^{-\Omega/T}+1)},\frac{(c_3q_R^2)^2}{(e^{-\Omega/T}+1)^2}]$ depends on both the coefficients c_i of the states in Eq. (8) and the Unruh temperature. Besides, according to the definition of geometric discord in Ref. [35, 36], we can also get the geometric discord from Eq. (10) only by changing min with max, so it is no doubt that the MIN is always greater than or equal to the geometric discord. And it should be note that the geometric discord with $c_i=1$ and $q_R=1$ will go back to the case in Ref. [34] which discussed the geometric discord by using the single-mode approximation.

For X-type initial state, the MIN gives different dynamic behaviors for different c_i:

(i) If $|c_1|,|c_2|\geq|c_3q_R^2|$ in Eq. (8), the minimum term in Eq. (10) is $\frac{(c_3q_R^2)^2}{(e^{-\Omega/T}+1)^2}$. In this case, the MIN, for fixed c_3 and q_R, decreases monotonously as the Unruh temperature increases.

(ii) For the case of $|c_3q_R^2|>\min\{|c_1|,|c_2|\}$ and both c_1 and c_2 don't equals to 0 at the same time, if $\min\{|c_1|,|c_2|\}\geq\frac{\sqrt{2}}{2}|c_3q_R^2|$, the MIN has a peculiar dynamics with a sudden change as the Unruh temperature increases, i.e., $N(\rho_{A,I})$ decays quickly until

$$T_{sc}=\frac{-\Omega}{\ln(\frac{|c_3q_R|^2}{min\{|c_1|^2,|c_2|^2\}}-1)}, \tag{11}$$

and then $N(\rho_{A,I})$ decays relatively slowly. Otherwise, the MIN decays monotonously as the temperature increases.

(iii) Finally, if $|c_1| = |c_2| = 0$, we have a monotonic decay of $N(\rho_{A,I})$.

The decrease of the MIN means that the difference between the pre- and post-measurement states becomes smaller, i.e., the disturbance induced by local measurement weaken. If we understand the MIN as some kind of correlations, this decrease means that the quantum correlation shared by two relatively accelerated observers decreases, i.e., less quantum resource can be used for the quantum information task by these two observers. So the Unruh effect affects quantum communication process by inducing the decrease of quantum resource.

By taking $\Omega = 1$ hereafter, we show the dynamical behavior of $N(\rho_{A,I})$ in Fig. 1 and 2. Fig. 1 exhibits one example of case (i) for different q_R, obviously, smaller q_R corresponds to less MIN, but it is no fundamental difference in the degradation of MIN for different choices of q_R. Fig 2 tells us some details of the sudden change occurring for case (ii), and the smaller q_R is, the earlier the sudden change occurs. Furthermore, We find that the MIN, as the Unruh temperature approaches to the infinite, has a limit

$$\lim_{T\to\infty} N(\rho_{A,I}) = \frac{1}{16}\{2(c_1 q_R)^2 + 2(c_2 q_R)^2 + (c_3 q_R^2)^2 - \min[2(c_1 q_R)^2, 2(c_2 q_R)^2, (c_3 q_R^2)^2]\}. \quad (12)$$

That is to say, as long as the initial MIN does not equal to zero, it can persist for arbitrary Unruh temperature.

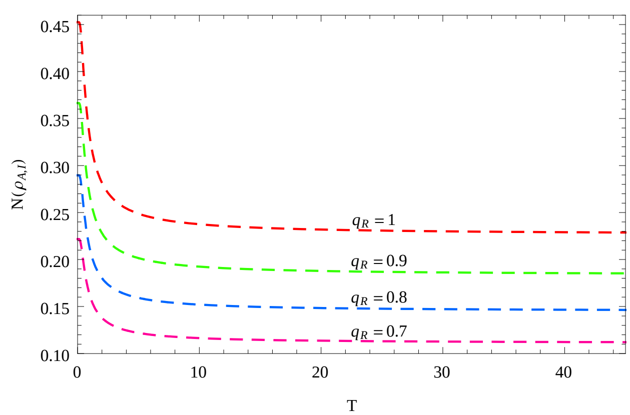

FIG. 1: (Color online) The MIN of state $\rho_{A,I}$ as a function of Unruh temperature T for different choices of q_R. We take parameters $c_1 = 1$, $c_2 = 0.9$ and $|c_3 q_R^2| \le |c_1|, |c_2|$ here.

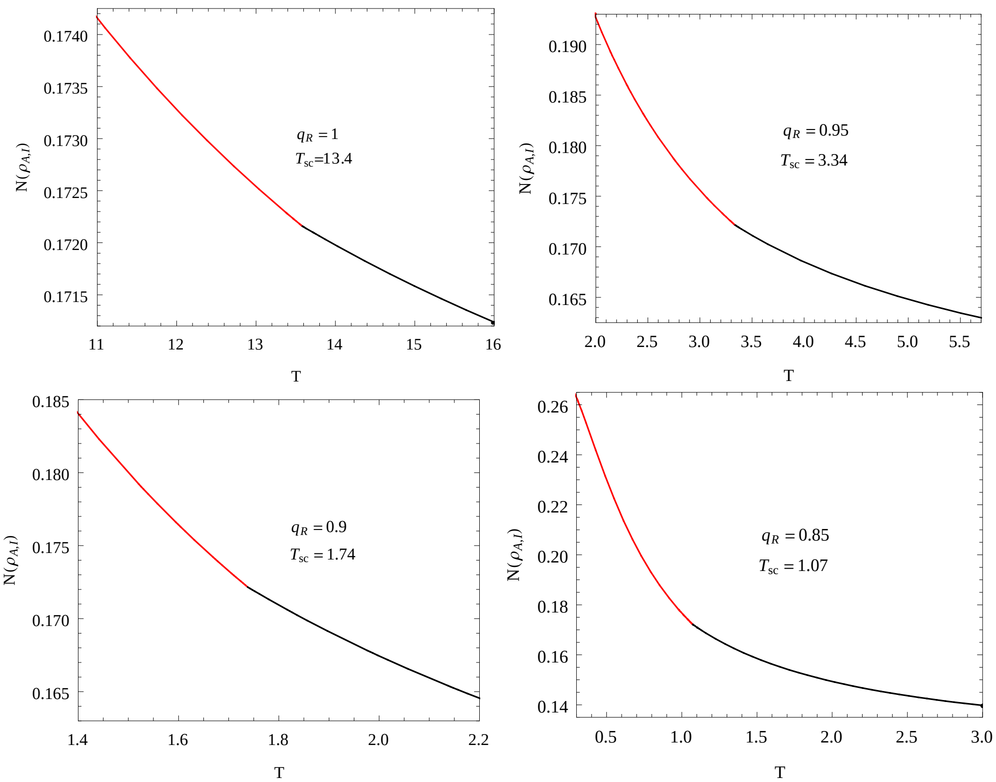

FIG. 2: (Color online) The details of sudden change of MIN, and the joint point between red line and black line is the sudden change point. Here we take $c_1 = 0.9$, $c_2 = 0.72$, and $c_3 = 1$.

We now study T_{sc} of Eq. (11). If $|c_1| \leq |c_2|$, by taking fixed c_3 and q_R, we plot how the parameter c_1 affects it in Fig. 3, which shows that it decreases monotonously as c_1 increases. That is to say, the bigger c_1 is, the sudden change behavior occurs earlier. Besides, q_R doesn't lead to the fundamental difference in the degradation of T_{sc} with the increase of c_1, but different q_R will result in different area of c_1 in which that sudden change can happen. And when $|c_2| \leq |c_1|$, it is interesting to note that with the increase of c_2 it decreases monotonously too.

B. MIN shared by Alice and Anti-Rob

Then we consider the MIN between modes A and antiparticle modes in region II. By tracing over all modes in region I and particle modes in region II, we get the reduced density matrix of Alice-antiRob modes

$$\rho_{A,II} = \frac{1}{4}\left(\mathbf{1}_A \otimes \mathbf{1}_{II} + c_0'\mathbf{1}_A \otimes \sigma_3^{(II)} + \sum_{i=1}^{3} c_i'\sigma_i^{(A)} \otimes \sigma_i^{(II)}\right), \tag{13}$$

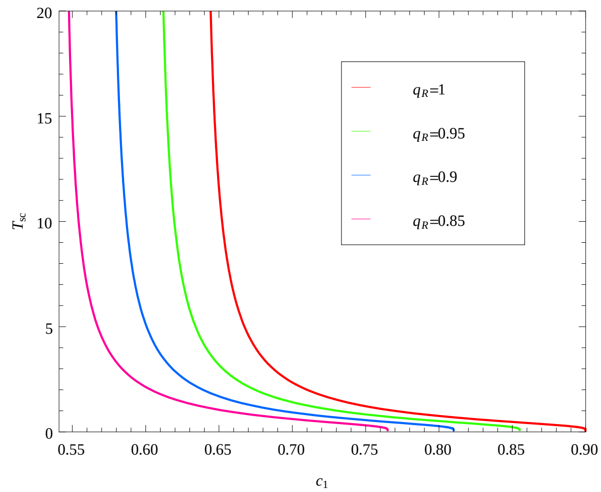

FIG. 3: (Color online) The T_{sc} as a function of c_1, here we take $|c_1| \leq |c_2|$ and $c_3 = 0.9$.

where $c_0' = \frac{e^{\Omega/T} - q_R^2}{(e^{\Omega/T}+1)}$, $c_1' = \frac{c_1 q_R}{(e^{\Omega/T}+1)^{\frac{1}{2}}}$, $c_2' = \frac{-c_2 q_R}{(e^{\Omega/T}+1)^{\frac{1}{2}}}$ and $c_3' = \frac{-c_3 q_R^2}{(e^{\Omega/T}+1)}$. Similarly, the MIN of state $\rho_{A,II}$ can be obtained according to Eq. (3), which is

$$
\begin{aligned}
N(\rho_{A,II}) = \frac{1}{4}\{ & \frac{(c_1 q_R)^2}{(e^{\Omega/T}+1)} + \frac{(c_2 q_R)^2}{(e^{\Omega/T}+1)} + \frac{(c_3 q_R^2)^2}{(e^{\Omega/T}+1)^2} \\
& - \min[\frac{(c_1 q_R)^2}{(e^{\Omega/T}+1)}, \frac{(c_2 q_R)^2}{(e^{\Omega/T}+1)}, \frac{(c_3 q_R^2)^2}{(e^{\Omega/T}+1)^2}]\}. \qquad (14)
\end{aligned}
$$

We can also obtain the geometric discord of Alice-antiRob modes by changing min with max in Eq. (14). Apparently, it is always smaller than or equal to the MIN.

By analysing Eq. (14), we will find that:

(i) If $|c_1|, |c_2| \geq |c_3 q_R^2|$, the MIN increases monotonously as the Unruh temperature increases for fixed c_3 and q_R.

(ii) For the case of $|c_3 q_R^2| > \min\{|c_1|, |c_2|\}$ and both c_1 and c_2 don't equal to 0 at the same time, if $\min\{|c_1|, |c_2|\} \leq \frac{\sqrt{2}}{2}|c_3 q_R^2|$, the MIN has a peculiar dynamics with a sudden change at T_{sc}

$$
T_{sc} = \frac{\Omega}{\ln(\frac{|c_3 q_R|^2}{min\{|c_1|^2, |c_2|^2\}} - 1)}. \qquad (15)
$$

Otherwise, the MIN increases monotonously with the increase of the Unruh temperature.

(iii) Finally, if $|c_1| = |c_2| = 0$, we have a monotonic increase of $N(\rho_{A,II})$.

In Fig. 4 and 5 we plot $N(\rho_{A,II})$ versus the Unruh temperature. And Fig. 4 shows one example of case (i), we find that the MIN, as the Unruh temperature increases, increases

monotonously, and as the Unruh temperature approaches to the infinite limit, it is close to

$$\lim_{T\to\infty} N(\rho_{A,II}) = \frac{1}{16}\{2(c_1q_R)^2 + 2(c_2q_R)^2 + (c_3q_R^2)^2 - \min[2(c_1q_R)^2, 2(c_2q_R)^2, (c_3q_R^2)^2]\}, \quad (16)$$

which is the same as $\lim_{T\to\infty} N(\rho_{A,I})$. In addition, as $T = 0$ the MIN vanishes, which means that the correlation of Alice-antiRob modes is local when the observers are inertial. Obviously, bigger q_R corresponds to bigger MIN. And Fig. 5 tells us that the sudden change, for fixed c_i, comes later for smaller q_R.

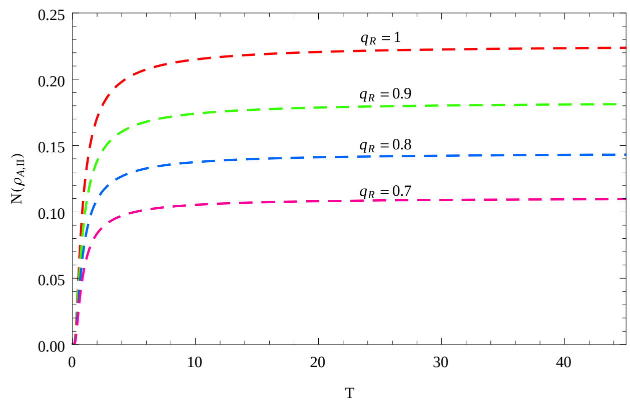

FIG. 4: (Color online) The MIN of state $\rho_{A,II}$ as a function of Unruh temperature T. We take parameters $c_1 = 1$, $c_2 = 0.9$ and $|c_3q_R^2| \le |c_1|, |c_2|$ with different q_R.

We study T_{sc} of Eq. (15), when $|c_1| < |c_2|$, for fixed c_3 and q_R, we plot T_{sc} as a function of c_1 in Fig. 6. We learn from the figure that, unlike the Fig. 3, T_{sc} increases monotonously with the increase of c_1. That is to say, the bigger c_1 is, the sudden change behavior occurs latter. Furthermore, q_R can change the area of c_1 in which that sudden change occurs. And when $|c_2| < |c_1|$, it is also important to note that as $|c_2|$ increases T_{sc} increases monotonously too.

C. Relating MIN to Nonlocality

Because the MIN is introduced to describe non-locality, for further understanding it we will compare it with the maximal possible value $\langle B_{max}\rangle$ of the Bell-CHSH expectation value.

As shown in Ref. [21], the $\langle B_{max}\rangle$ for a given state ρ is determined by

$$\langle B_{max}\rangle_\rho = 2\sqrt{\mu_1 + \mu_2}, \quad (17)$$

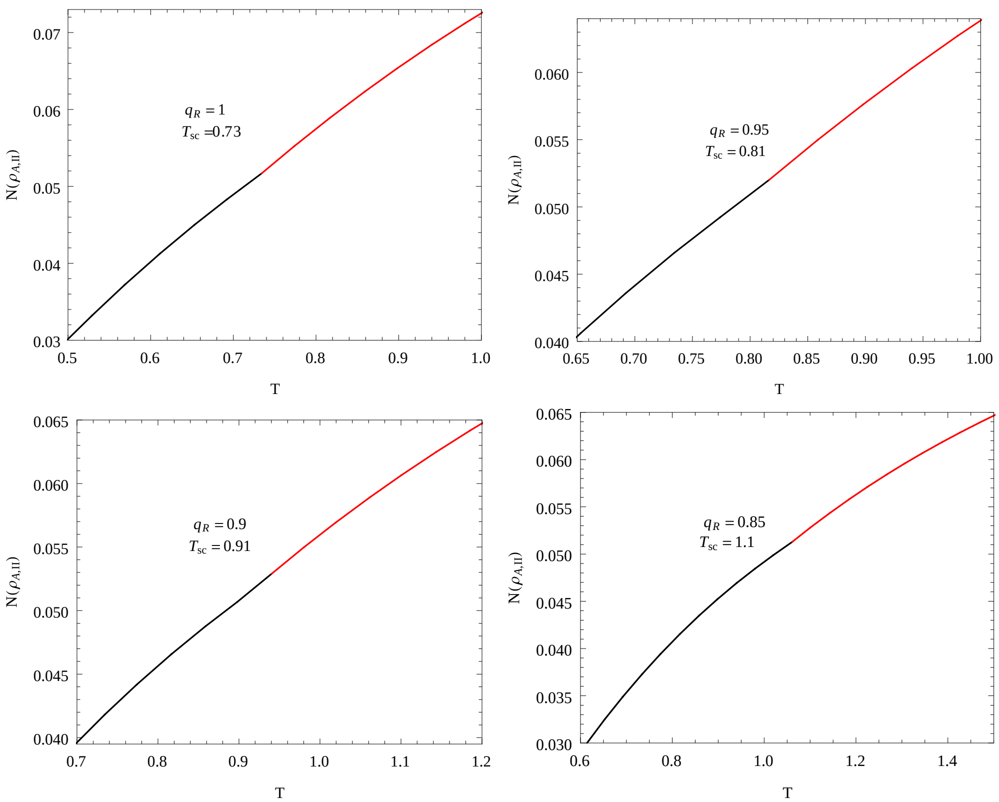

FIG. 5: (Color online) The details of sudden change of MIN for state $\rho_{A,II}$, and the joint point between red line and black line is the sudden change point. Here we take $c_1 = 0.9$, $c_2 = 0.45$, and $c_3 = 1$.

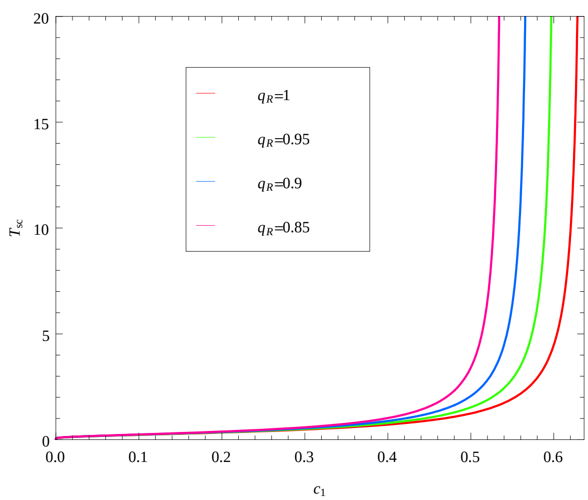

FIG. 6: (Color online) The T_{sc} as a function of c_1, here we take $|c_1| \leq |c_2|$ and $c_3 = 0.9$.

where μ_1, μ_2 are the two largest eigenvalues of $U(\rho) = TT^t$, the matrix $T = (t_{ij})$ with $t_{ij} = Tr[\rho\sigma_i \otimes \sigma_j]$.

Using Eqs. (9) and (17), $\langle B_{max}\rangle_{\rho_{A,I}}$ is given by

$$\begin{aligned}\langle B_{max}\rangle_{\rho_{A,I}} = 2\{&\frac{(c_1 q_R)^2}{(e^{-\Omega/T}+1)} + \frac{(c_2 q_R)^2}{(e^{-\Omega/T}+1)} + \frac{(c_3 q_R^2)^2}{(e^{-\Omega/T}+1)^2} \\ &- \min[\frac{(c_1 q_R)^2}{(e^{-\Omega/T}+1)}, \frac{(c_2 q_R)^2}{(e^{-\Omega/T}+1)}, \frac{(c_3 q_R^2)^2}{(e^{-\Omega/T}+1)^2}]\}^{1/2}, \end{aligned} \tag{18}$$

It is interesting to note that

$$N(\rho_{A,I}) = \frac{1}{16}\langle B_{max}\rangle^2_{\rho_{A,I}}. \tag{19}$$

We plot $N(\rho_{A,I})$ versus $\langle B_{max}\rangle_{\rho_{A,I}}$ in Fig. 7, which shows that $N(\rho_{A,I})$ increases monotonously as $\langle B_{max}\rangle_{\rho_{A,I}}$ increases and it vanishes at zero point of $\langle B_{max}\rangle_{\rho_{A,I}}$. It is well known that Bell inequality must be obeyed by local realism theory, but may be violated by quantum mechanics. If we get $\langle B_{max}\rangle_{\rho_{A,I}} > 2$, it means that the violation of Bell-CHSH inequality, which tells us that there exists nonlocal quantum correlation. But when $\langle B_{max}\rangle_{\rho_{A,I}} \leq 2$, it doesn't mean that no quantum correlation exists, at least for some mixed states, which have quantum correlation but obey the Bell inequality. So we can't be sure that whether quantum correlation exists or not when $\langle B_{max}\rangle_{\rho_{A,I}} \leq 2$. However, the MIN, which is an indicator of the global effect caused by locally invariant measurement, is introduced to quantify nonlocality, nonzero MIN means existence of nonlocality. And from Fig. 7 we see that the MIN persists for all $\langle B_{max}\rangle_{\rho_{A,I}}$ except for zero. Thus, the MIN, understood as some kind of correlations, is more general than the quantum nonlocality related to violation of the Bell's inequalities.

IV. MIN FOR BOSONIC FIELDS

For Bosonic field, we can also get the Unruh vacuum in terms of Rindler vacuum and its excitations. As shown in Ref. [8], the monochromatic Unruh vacuum mode can be expressed as

$$|0_\Omega\rangle_U = \frac{1}{\cosh r_\Omega}\sum_{n=0}^{\infty} \tanh^n r_\Omega |n_\Omega\rangle_{\mathrm{I}} |n_\Omega\rangle_{\mathrm{II}}, \tag{20}$$

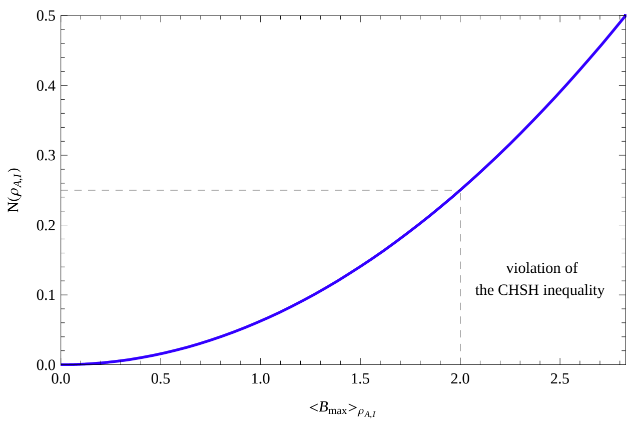

FIG. 7: (Color online) The MIN of state $\rho_{A,I}$ as function of the maximally possible value of the Bell-CHSH expectation value.

with $\tanh r_\Omega = e^{-\frac{\Omega}{2T}}$. And a positive frequency particle state is given by

$$\begin{aligned} A^\dagger_{\Omega,\mathrm{U}}|0_\Omega\rangle_\mathrm{U} &= |1_\Omega\rangle_\mathrm{U} \\ &= \sum_{n=0}^{\infty} \frac{\tanh^n r_\Omega}{\cosh r_\Omega} \frac{\sqrt{n+1}}{\cosh r_\Omega} (q_R|(n+1)_\Omega\rangle_\mathrm{I}|n_\Omega\rangle_\mathrm{II} + q_L|n_\Omega\rangle_\mathrm{I}|(n+1)_\Omega\rangle_\mathrm{II}). \end{aligned} \tag{21}$$

For simplicity, we choose the Bell state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0_\omega\rangle_\mathrm{M}|0_\Omega\rangle_\mathrm{U} + |1_\omega\rangle_\mathrm{M}|1_\Omega\rangle_\mathrm{U}) \tag{22}$$

as initial state and study the MIN of both Alice-Rob modes and Alice-antiRob modes. After rewriting Rob's modes in Eq. (22) in terms of the Rindler basis, the Alice-Rob density matrix is obtained by tracing over region II, with the result,

$$\rho_{A,I} = \frac{1}{2}\sum_{n=0}^{\infty}\left[\frac{\tanh^n r_\Omega}{\cosh r_\Omega}\right]^2 \rho^n_{A,R}, \tag{23}$$

where

$$\begin{aligned} \rho^n_{A,R} = \ & |0n\rangle\langle 0n| + \frac{n+1}{\cosh^2 r_\Omega}(|q_R|^2|1n+1\rangle\langle 1n+1| + |q_L|^2|1n\rangle\langle 1n|) \\ & + \frac{\sqrt{n+1}}{\cosh r_\Omega}(q_R|1n+1\rangle\langle 0n| + q_L\tanh r_\Omega|1n\rangle\langle 0n+1|) + \frac{\sqrt{(n+1)(n+2)}}{\cosh^2 r_\Omega} \\ & \times q_R q_L^* \tanh r_\Omega |1n+2\rangle\langle 1n| + (\mathrm{H.c.})_{\mathrm{nondiag}}, \end{aligned}$$

where $(\mathrm{H.c.})_{\mathrm{nondiag}}$ means Hermitian conjugate of only the non-diagonal terms. And it is necessary to note that the Alice-antiRob density matrix, due to the symmetry in the Unruh modes between regions I and II, can be gained by exchanging q_R and q_L.

It is convenient to rewrite state (23) in the following form,

$$\rho_{A,I} = \frac{1-t^2}{2}(|0\rangle\langle 0| \otimes M_{00} + |1\rangle\langle 1| \otimes M_{11} + |0\rangle\langle 1| \otimes M_{01} + |1\rangle\langle 0| \otimes M_{10}), \tag{24}$$

where $t = \tanh r_\Omega$ and the matrices on Rob's Hilbert space are

$$\begin{aligned}
M_{00} &= \sum_{n=0}^{\infty} t^{2n}|n\rangle\langle n|, \\
M_{11} &= (1-t^2)\sum_{n=0}^{\infty} t^{2n}[(n+1)(|q_R|^2|n+1\rangle\langle n+1| + |q_L|^2|n\rangle\langle n|) \\
&\quad + t\sqrt{(n+1)(n+2)}(q_R q_L^*|n+2\rangle\langle n| + q_R^* q_L|n\rangle\langle n+2|)], \\
M_{01} &= \sqrt{1-t^2}\sum_{n=0}^{\infty}\sqrt{n+1}t^{2n}(q_R^*|n\rangle\langle n+1| + q_L^*|n+1\rangle\langle n|), \\
M_{10} &= M_{01}^{\dagger}.
\end{aligned}$$

We make a projective measurement on the qubit of Alice, and after the measurement the final state is given by

$$\rho'_{A,I} = \sum_{\alpha=\pm}(\Pi_\alpha \otimes \mathbf{1}_I)\rho_{A,I}(\Pi_\alpha \otimes \mathbf{1}_I) = \sum_{\alpha=\pm} p_\alpha \Pi_\alpha \otimes \rho_{R|\alpha}. \tag{25}$$

Here, the projectors

$$\Pi_\pm = \frac{1}{2}[(1 \pm x_3)|0\rangle\langle 0| + (1 \mp x_3)|1\rangle\langle 1| \pm (x_1 - ix_2)|1\rangle\langle 0| \pm (x_1 + ix_2)|0\rangle\langle 1|],$$

$\rho_{R|\alpha}$ denotes the post-measured state of Rob's reduced system conditioned on the outcome α, and p_α is the corresponding probability.

After a series of calculations, we finally get the difference between the pre- and post-measurement states, with the result,

$$\begin{aligned}
\mathrm{Tr}((\rho_{A,I} - \rho'_{A,I})^2) &= \frac{(1-t^2)^2}{8}[(1-x_3^2)(\mathrm{Tr}(M_{00}^2) + \mathrm{Tr}((M_{11}^2)) - 2\mathrm{Tr}(M_{00}M_{11})) \\
&\quad + 2(1+x_3^2)\mathrm{Tr}(M_{01}M_{10})] \\
&= \frac{(1-t^2)^2}{8}[\frac{1+x_3^2}{1+t^2} + \frac{2(1-t^2)(|q_R|^2 - |q_L|^2)}{1+t^2} \\
&\quad + \frac{1-x_3^2}{(1+t^2)^3}((1+t^4) - 2(1-t^2)^2|q_R|^2|q_L|^2)].
\end{aligned} \tag{26}$$

Therefore, according to the definitions of MIN [26] and geometric discord [35, 36], Eq. (26) with $x_3 = 1$ and $x_3 = 0$ gives the MIN and geometric discord, respectively. When $x_3 = 0$ and $q_R = 1$, our result can go back to the case in Ref. [34] where the geometric discord

is obtained by using the single-mode approximation. Furthermore, we can also obtain the MIN and geometric discord of Alice-antiRob modes by exchanging q_R and q_L in Eq. (26). And it is interesting to note that both the MIN and geometric discord approach to zero in the infinity limit of Unruh temperature ($t \to 1$), which is different from that of Dirac field case discussed above.

In Fig. 8, we plot the MIN and geometric discord as a function of parameterized Unruh temperature for Alice-Rob modes. It is shown that: (i) when $0 \leq q_R < q_{sc} = \frac{1}{\sqrt{6}}$ the MIN, as t increases, firstly increases to an maximal value, then decays monotonously, and finally approaches to zero. Interestingly, when $0 \leq q_R < q_{sc} \approx 0.37566$ the geometric discord has the same dynamic behavior too; and (ii) both the MIN and geometric discord, when $q_R \geq q_{sc}$, degrades monotonously with the increase of t. Obviously, the dynamic behavior (i) is distinctly different from Dirac fields case that the MIN always decays.

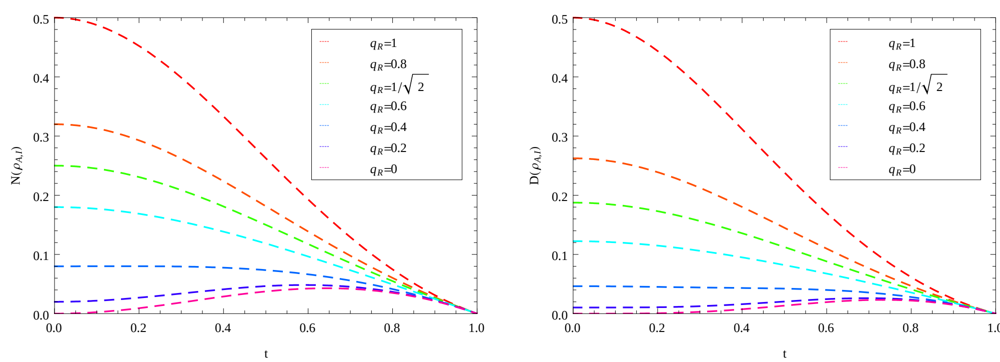

FIG. 8: (color online) The MIN (left one) and geometric discord (right one) as a function of parameterized Unruh temperature $t = e^{-\frac{\Omega}{2T}}$ with fixed q_R.

In Fig. 9, we plot the MIN and geometric discord as a function of parameterized Unruh temperature for Alice-antiRob modes. It is shown that: (i) when $0 \leq q_R \leq q_{sc} = \sqrt{\frac{5}{6}}$ the MIN degrades monotonously with the increase of t, and approaches to zero at the infinite acceleration limit. Interestingly, when $0 \leq q_R \leq q_{sc} \approx 0.92676$ the geometric discord has the same dynamic behavior too; (ii) both the MIN and geometric discord, when $q_R > q_{sc}$, firstly increases to an maximal value, then decays monotonously, and finally approaches to zero. And it is interesting to note that these dynamic behaviors are in sharp contrast to the Dirac fields case that the MIN always increases.

The obvious distinction between Dirac MIN and Bosonic MIN is directly caused by the differences between Fermi-Dirac and Bose-Einstein statistics. The Dirac particles must obey

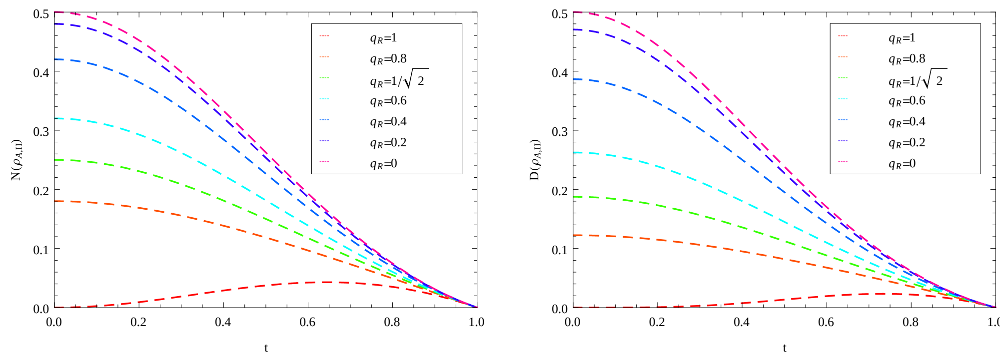

FIG. 9: (color online) The MIN (left one) and geometric discord (right one) as a function of parameterized Unruh temperature $t = e^{-\frac{\Omega}{2T}}$ with fixed q_R.

the Pauli exclusion principle and access only two quantum levels, while the Bosonic fields is the infinite dimensional system.

V. CONCLUSIONS

How the Unruh effect affects the MIN of Dirac and Bosonic fields was investigated, and the following properties were found.

For Dirac fields, we choose the X-type states as initial state, and find that: (i) the MIN $N(\rho_{A,I})$ decreases as the Unruh temperature increases, while $N(\rho_{A,II})$ is contrary to it. However, $\lim_{T\to\infty} N(\rho_{A,II}) = \lim_{T\to\infty} N(\rho_{A,I})$, and they always persists when $q_R \neq 0$; (ii) both $N(\rho_{A,I})$ and $N(\rho_{A,II})$ have a peculiar dynamics with a sudden change at T_{sc} provided c_i is appropriately chosen. The T_{sc} for $N(\rho_{A,I})$ decreases as c_i increases, while it is contrary for $N(\rho_{A,II})$ case; and (iii) the MIN is more general than the quantum nonlocality related to violation of Bell's inequalities. Besides, it is always equal or larger than the geometric discord.

For Bosonic field, we take the Bell state, which is a special case of the X-type state, as initial state and find that: (i) no matter for Alice-Rob modes or Alice-antiRob modes, both the MIN and geometric discord eventually approach to zero when $T \to \infty$, that is to say, no "quantum correlation" exists in infinite acceleration limit; (ii) the choice of q_R leads to two different dynamic behaviors of the MIN and geometric discord. One is that the quantity firstly increases to an maximal value, then decays monotonously, and finally approaches to zero, and the other is that the quantity degrades monotonously to zero with the increase of

t; and (iii) as a correlation, the MIN is always bigger than or equal to geometric discord.

A distinctly distinguishable property of the MIN in non-inertial frames is that the MIN between Dirac fields remains non-zero in the infinite acceleration limit, while for Bosonic fields it vanishes. Which indicates that the MIN between Dirac fields is more robust than Bosonic fields case when they undergo Unruh effect. Furthermore, for Dirac fields there is no fundamental difference of degradation (increase) of MIN between Alice-Rob modes (Alice-antiRob modes) when we takes different q_R. However, for Bosonic fields different q_R will lead to different dynamic behaviors of its MIN. Different statistics for Dirac and Bosonic fields directly result in these distinctions. Which suggests fundamental differences between infinite and finite dimensional system MIN in relativistic settings.

Our discussion can be extended to the background of black hole by assuming that Rob locates at the event horizon, while Alice freely falls into the black hole. Therefore, by replacing the Unruh temperature with Hawking temperature, our result shows that the MIN for Dirac fields persists in the infinite Hawking temperature limit, while that for Bosonic fields vanishes. It is believed that further investigation of relativistic quantum information may have implications for the problem of black hole information loss.

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[1] P. M. Alsing, and G. J. Milburn, Phys. Rev. Lett. **91**, 180404 (2003).

[2] P. M. Alsing I. Fuentes-Schuller, R. B. Mann, and T. E. Tessier, Phys. Rev. A. **74**, 032326 (2006).

[3] I. Fuentes-Schuller, and R. B. Mann, Phys. Rev. Lett. **95**, 120404 (2005).

[4] Y. Ling, S. He, W. G. Qiu, and H. B. Zh, Journal of Physics A Mathematical and Theoretical. **40**, 9025 (2007).

[5] S. Moradi, Phys. Rev. A. **79**, 064301 (2009).

[6] Q. Y. Pan, and J. L. Jing, Phys. Rev. A. **77**, 024302 (2008); Q. Y. Pan, and J. L. Jing, Phys. Rev. D. **78**, 065015 (2008); J. C. Wang, and J. L. Jing, Phys. Rev. A. **83**, 022314 (2011); J. C. Wang, and J. L. Jing, Annals of Physics. **2**, 327, 283-291 (2011).

[7] J. Leon, and E. Martín-Martínez, Phys. Rev. A. **80**, 012314 (2009).

[8] D. E. Bruschi, J. Louko, E. Martín-Martínez, A. Dragan, and I. Fuentes, Phys. Rev. A. **82**, 042332 (2010).

[9] M. Montero, and E. Martín-Martínez, Phys. Rev. A. **85**, 024301 (2012).

[10] M. Montero, and E. Martín-Martínez, Phys. Rev. A. **84**, 012337 (2011).

[11] J. C. Wang, J. F. Deng, and J. L. Jing, Phys. Rev. A. **81**, 052120 (2010); Z. H. Tian, and J. L. Jing, Phys. Lett. B. **707**, 264-271 (2012).

[12] A. Datta, Phys. Rev. A. **80**, 052304 (2009).

[13] M. X. Han, S. J. Olson, and J. P. Dowling, Phys. Rev. A. **78**, 022302 (2008).

[14] David C. M. Ostapchuk, and R. B. Mann, Phys. Rev. A. **79**, 042333 (2009).

[15] R. B. Mann, and V. M. Villalba, Phys. Rev. A. **80**, 022305 (2009).

[16] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).

[17] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki. Rev. Mod. Phys. **81**, 865 (2009).

[18] *The Physics of Quantum Information*, edited by D. Bouwmeester, A. Ekert, A. Zeilinger (Springer-Verlag, Berlin, 2000).

[19] L. Bombelli, R. K. Koul, J. Lee, and R. Sorkin. Phys. Rev. D **34**, 373 (1986).

[20] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975); Phys. Rev. D **14**, 2460 (1976); H. Terashima, Phys. Rev. D **61**, 104016 (2000).

[21] N. Friis, P. Köhler, E. Martín-Martínez, and R. A. Bertlmann. Phys. Rev. A. **84**, 062111 (2011).

[22] A. Smith, and R. B. Mann. arxiv: 1107. 4633.

[23] D. Park, arXiv:1201. 1335.

[24] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880-884 (1969).

[25] G. Syetlichny, Phys. Rev. D. **35**, 3066 (1987).

[26] Sh. L. Luo, and Sh. Sh. Fu, Phys. Rev. Lett. **106**, 120401 (2011).

[27] Zh. J. Xi, X. G. Wang, and Y. M. Li, arxiv:1112. 0736.

[28] S. Y. Mirafzali, I. Sargolzahi, A. Ahanj, K. Javidan, and M. Sarbishaei, arxiv:1110. 3499.

[29] A. Sen, D. Sarkar, and A. Bhar, arxiv:1201. 2041.

[30] Y. Guo, and J. Ch. Hou, arxiv:1107. 0355.

[31] M. L. Hu, and H. Fan, arxiv:1201. 6430.

[32] G. F. Zhang, H. Fan, Ai. L. Ji, and W. M. Liu, arxiv:1201. 1949.

[33] W. G. Unruh, Phys. Rev. D. **14**, 870 (1976).

[34] Eric G. Brown, Kyle Cormier, Eduardo Martín-Martínez, Robert B. Mann, arXiv:1205.1268.

[35] B. Dakić, V. Vedral, and Č. Brukner, Phys. Rev. Lett. **105**, 190502 (2010).

[36] D. Girolami, and G. Adesso, Phys. Rev. A. **83**, 052108 (2011).